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Hewlett Packard Labs
HPE-2016-13

Keyword(s):
Vertical cavity surface emitting lasers; Electro-optical devices; Photonic integrated circuits; Optical interconnects

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Directly-modulated vertical-cavity surface-emitting lasers (VCSELs) are commonly used in short-reach optical interconnect applications. In order to enable efficient optical interconnect transceiver systems operating at data rates in excess of 20Gb/s, co-simulation environments which allow for the optimization of driver circuitry with accurate compact VCSEL models are necessary. This paper presents a compact comprehensive Verilog-A VCSEL model which captures thermally-dependent electrical and optical dynamics and provides dc, small signal, and large-signal simulation capabilities. The device’s electrical behavior is described with an equivalent circuit which captures both large-signal operation and electrical parasitics, while the optical response is captured with a rate-equation-based model, with bias and temperature dependencies incorporated into key electrical and optical model parameters. Experimental verification of the model is performed at 25Gb/s with a 990nm VCSEL to study the impact of bias current level and substrate temperature.
A comprehensive Verilog-A VCSEL model for >20Gb/s optical interconnect transceiver circuit design

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Abstract: Directly-modulated vertical-cavity surface-emitting lasers (VCSELs) are commonly used in short-reach optical interconnect applications. In order to enable efficient optical interconnect transceiver systems operating at data rates in excess of 20Gb/s, co-simulation environments which allow for the optimization of driver circuitry with accurate compact VCSEL models are necessary. This paper presents a compact comprehensive Verilog-A VCSEL model which captures thermally-dependent electrical and optical dynamics and provides dc, small signal, and large-signal simulation capabilities. The device’s electrical behavior is described with an equivalent circuit which captures both large-signal operation and electrical parasitics, while the optical response is captured with a rate-equation-based model, with bias and temperature dependencies incorporated into key electrical and optical model parameters. Experimental verification of the model is performed at 25Gb/s with a 990nm VCSEL to study the impact of bias current level and substrate temperature.

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OCIS codes: (250.7260) Vertical cavity surface emitting lasers; (230.2090) Electro-optical devices; (250.5300) Photonic integrated circuits; (200.4650) Optical interconnects.

References and links

1. Introduction

In order to support the bandwidth demands of future data centers and supercomputers, it is essential to improve the data rate, energy efficiency, and cost of the optical interconnects employed in these systems [1, 2]. Vertical-cavity surface-emitting laser (VCSEL)-based optical interconnects [3-7] are well suited for these applications [8, 9] due to their simple direct modulation, excellent energy efficiency, high data rate, and low-cost packaging. Potential solutions to address the growing link distances in these systems include the use of advanced modulation schemes, such as four-level pulse-amplitude modulation (PAM4) [10, 11] and discrete multitone [12], and/or single-mode VCSELs [13-16].

VCSEL bandwidth, which is bias and temperature dependent, is limited by a combination of electrical parasitics and the electron-photon interaction described by a set of second-order rate equations [17]. This results in optical interconnect systems often incorporating equalization circuitry, which may be non-linear, embedded in either the transmitter drivers or optical front-ends to extend the data rate [3-7]. A VCSEL’s large-signal temperature-dependent static optical power-current-voltage (L-I-V) response is also important, particularly
for advanced modulation schemes where linearity is a concern, such as PAM4. Due to these non-linear dynamics and thermal dependencies of VCSELs, transmitter circuity must be carefully designed to supply the required signals at high data rates with relatively low-power consumption. This motivates co-simulation environments with a compact comprehensive VCSEL model that captures thermally-dependent electrical and optical dynamics and provides dc, small signal, and large-signal simulation capabilities.

While numerous VCSEL models have been developed, some of the previous models have not included thermal effects [18-22]. Other work which has included temperature effects have neglected either bias and temperature-dependent electrical parasitics [23-27] or the rate-equation-based electrical-to-optical conversion dynamics [28]. This paper presents a compact Verilog-A VCSEL model which comprehensively captures the large- and small-signal dynamic response combined with the dc thermal effects on both the electrical and optical behavior. The model is described in Section II and consists of an accurate large- and small-signal electrical input stage coupled to a rate-equation-based optical output stage formulated for efficient Verilog-A implementation. To provide a more intuitive model for the case of a VCSEL cavity, the general laser rate equations have been reformulated. Section III presents 25Gb/s experimental verification of the VCSEL model, which was performed both at 23°C and 80°C for varying bias current levels. Finally, Section IV concludes the paper.

### 2. Model description and parameter extraction

Figure 1 shows both a typical VCSEL cross section view with the equivalent electrical small-signal model placed where the circuit elements physically originate and a general laser cavity structure. This electrical model consists of a parasitic capacitor \( C_p \) between the laser diode’s anode and cathode terminals, a resistor \( R_m \) that models the p- and n-type distributed Bragg reflectors (DBRs), and the active region resistance \( R_a \) and capacitance \( C_a \). While this model is suitable for small-signal behavior, an accurate comprehensive model requires inclusion of both bias- and thermal-dependent large-signal and optical dynamics formed by the interaction of the active region carriers \( N_c \) and the cavity power \( P_c \) and key gain and loss parameters.

Figure 2 shows the proposed VCSEL model which consists of an electrical input stage and a rate-equation-based optical output stage. When a driving current is applied to the electrical input stage, large-signal behavior is captured with the inclusion of the bias-dependent voltage source \( V_{dc} \). Linking the electrical and optical stages is the active region current through \( R_a \), which serves as the key input to the carrier rate equation. The optical stage’s two coupled differential equations describe the dynamic carrier and photon interaction to capture the nonlinear transient behavior of the optical output power \( P_{opt} \). Both the electrical and optical stages have thermally-dependent elements to model how the VCSEL’s performance varies with temperature. All circuit elements in the electrical stage are functions of both substrate temperature \( T_s \) and bias current \( I_b \), except for \( C_p \) which is modeled as being only temperature dependent due to its negligible change with bias current. While in the optical stage, the injection efficiency \( \eta_i \), threshold current \( I_{th} \), round trip cavity gain \( \delta_g \), and spontaneous emission power \( P_{sp} \) are modeled as temperature dependent.

![Fig. 1](image-url)  
(a) VCSEL cross section view with the equivalent electrical small-signal model overlaid, (b) General laser cavity structure.
The model parameters are extracted from curve fitting measurements of VCSEL dc, small-signal, and large-signal dynamic behavior, with polynomial-based curve fitting employed to allow for an efficient realization in the Verilog-A model. Both the large-signal voltage-current (V-I) characteristic and the small-signal S11 data are utilized to extract the electrical model parameters. While the optical model parameters are extracted from the VCSEL’s physical design, large-signal optical power-current (L-I) characteristic, small-signal S21 data, and high-speed pulse response measurements.

2.1 DC characteristics

Since VCSELs are operated above the threshold current in high data rate optical interconnect systems, the temperature-dependent optical power-current-voltage (L-I-V) characteristics are modeled as

\[
V(T) = V_{on} + R_d(T) \cdot I = V_{dc}(T_s, I_b) + \left[ R_m(T_s, I_b) + R_d(T_s, I_b) \right] \cdot I
\]

(1)

\[
P_{opt}(T) = \eta(T) \cdot \left[ I - I_{th}(T) \right]
\]

(2)

where \( V_{on} \) is the turn-on lasing voltage, \( R_d \) is the differential resistance, and \( \eta \) is the slope efficiency. These characteristics are functions of the active region temperature \( T \), which is

\[
T = T_s + (VI - P_{opt}) \cdot R_{th}
\]

(3)

where \( R_{th} \) is the thermal resistance. This can be measured indirectly through the lasing wavelength [29] by

\[
R_{th} = \frac{\Delta T}{\Delta P_{th}} = \frac{\Delta \lambda_{th}}{\Delta P_{th}} \cdot \frac{\Delta P_{th}}{\Delta T}
\]

(4)

where the heat power is

\[
P_{th} = VI - P_{opt}
\]

(5)

and \( \Delta \lambda / \Delta P_{th} \) and \( \Delta \lambda / \Delta T \) are the wavelength change with heat power and substrate temperature, respectively. Utilizing an optical spectrum analyzer (OSA) to measure a 990nm 25Gb/s-class VCSEL’s fundamental mode wavelength change with heat power and substrate temperature yields values of \( \Delta \lambda / \Delta P_{th} = 0.159 \text{nm/mW} \) and \( \Delta \lambda / \Delta T = 0.070 \text{nm/oC} \), and therefore \( R_{th} = 2.27 \text{oC/mW} \).

In order to obtain the remaining parameters for the dc characteristics, \( \eta \) is first decomposed into parameters used in the rate-equation-based optical output stage

\[
\eta(T) = \eta(T) \cdot \frac{\delta_{th}}{\delta_{th} + \delta_{a}} \cdot \frac{h \nu_{th}}{q}
\]

(6)
where $\eta_i$ is the injection efficiency, $\delta_1=1-R_1$ is the output mirror (DBR) transmissivity, $R_1$ is the reflectivity, $\delta_a$ is the round trip internal absorption loss, $h\nu_0$ is the photon energy, and $q$ is the electron charge. By curve fitting the Figure 3 measured L-I-V characteristics, the three parameters $R_d$, $\delta_a$, and $I_{th}$ are then extracted as a polynomial function of temperature.

$$f(T) = b_0 + b_1 \cdot (T - T_0) + b_2 \cdot (T - T_0)^2$$

Table 1 gives the $b_0$-$b_2$ are coefficients, with $\delta_1=3.8 \times 10^{-3}$, $\delta_a=2.7 \times 10^{-3}$, and $V_{cm}=1.28$V assumed fixed and a room temperature $T_0=23^\circ$C is utilized. Excellent matching is achieved with the measured Figure 3 characteristics, with the differential resistance and slope efficiency decreasing with substrate temperature and the VCSEL’s thermal rollover point appearing near a bias of 14mA at 80°C.

![Fig. 3. Measured and simulated (a) V-I and (b) L-I characteristics.](image)

**Table 1. Polynomial coefficients of $R_d$, $\delta_a$, and $I_{th}$.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
</tr>
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<tbody>
<tr>
<td>$R_d$</td>
<td>Ω</td>
<td>137.0</td>
<td>-0.4836</td>
<td>0.0018</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>-</td>
<td>0.90</td>
<td>1.80×10^{-3}</td>
<td>-8.54×10^{-6}</td>
</tr>
<tr>
<td>$I_{th}$</td>
<td>mA</td>
<td>0.30</td>
<td>1.79×10^{-3}</td>
<td>4.20×10^{-3}</td>
</tr>
</tbody>
</table>

2.2 Small-signal characteristics

The remaining parameters in the electrical model and the small-signal response of the optical model are extracted by curve fitting the Figure 4 small-signal electrical input impedance and VCSEL magnitude transfer function measurements. As these responses are functions of the large-signal operating point, responses for three different bias points corresponding to the nominal 6mA swing centered at a 6mA bias level are utilized.

In order to accurately describe the VCSEL’s electrical parasitic elements dependency on bias and temperature, $R_m$, $R_o$, and $C_p$ are modeled as a third-order function of bias current with first-order coefficients which change with substrate temperature.

$$Z(T_s, I_s) = \left[c_0 + c_1 \cdot (T_s - T_0)\right] + \left[c_2 + c_3 \cdot (T_s - T_0)\right] \cdot (I_s - I_0)$$

$$+ \left[c_4 + c_5 \cdot (T_s - T_0)\right] \cdot (I_s - I_0)^2 + \left[c_6 + c_7 \cdot (T_s - T_0)\right] \cdot (I_s - I_0)^3$$

As $C_p$ is a weak function of the bias current, it is modeled as having only a first-order dependency on substrate temperature. Table 2 gives the $c_0$-$c_7$ coefficients for $T_0=23^\circ$C and $I_0=6$mA. The electrical model is completed by utilizing the $R_m$ and $R_o$ values at a given temperature and bias to compute the dc voltage source $V_{dc}$ from Eq. (1), providing flexibility to match both the large signal and small-signal characteristics. Excellent matching is achieved...
for both the real and imaginary components of the measured Figure 4 electrical input impedance data, with the modeled response achieving the expected low-pass characteristic at both 23°C and 80°C.

![Fig. 4. Measured and simulated electrical input impedance at (a) 23°C (room temperature) and (b) 80°C substrate temperatures, and normalized optical magnitude transfer response at (c) 23°C and (d) 80°C substrate temperatures.](image)

**Table 2. Polynomial coefficients of $R_m$, $R_a$, $C_a$, and $C_p$.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$c_5$</th>
<th>$c_6$</th>
<th>$c_7$</th>
</tr>
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<tbody>
<tr>
<td>$R_m$</td>
<td>Ω</td>
<td>49.408</td>
<td>-0.048</td>
<td>-88.663</td>
<td>-0.106</td>
<td>-8.127×10^4</td>
<td>-634.185</td>
<td>-6.947×10^6</td>
<td>4.392×10^6</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Ω</td>
<td>67.936</td>
<td>-0.290</td>
<td>-2.319×10^3</td>
<td>22.030</td>
<td>2.830×10^4</td>
<td>-324.011</td>
<td>8.817×10^4</td>
<td>4.714×10^3</td>
</tr>
<tr>
<td>$C_a$</td>
<td>fF</td>
<td>114.1</td>
<td>0.735</td>
<td>1.84×10^4</td>
<td>-17.47</td>
<td>3.70×10^6</td>
<td>-521.6</td>
<td>-1.981×10^7</td>
<td>2.228×10^5</td>
</tr>
<tr>
<td>$C_p$</td>
<td>fF</td>
<td>20</td>
<td>-0.175</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

A two-pole transfer function, derived from the VCSEL rate equations, models the optical small-signal response

$$H(f,T) = \frac{1}{1 - \left[ \frac{f}{f_r(T)} \right]^2 + j \cdot \left[ \frac{f}{f_r(T)} \right]^2 \left[ f / f_d(T) \right]},$$

(9)

where $f_r$ is the resonance frequency and $f_d$ is the damping frequency.

$$f_r(T) = D(T) \cdot \sqrt{I - I_{th}(T)}$$

(10)
Expressing the small-signal transfer function in this manner allows easy identification of the quality factor $Q_e = f_r / f_d$, which is useful in the analysis of relative intensity noise (RIN) and modulation overshoot. In the $f_r$ and $f_d$ equations, the $D$-factor quantifies the resonance frequency increase with current, the $K$-factor defines the VCSEL’s intrinsic modulation bandwidth capabilities, $γ$ is the damping factor and $γ_0$ is the damping factor offset [17]. These $D$- and $K$- factors are modeled as second-order functions of temperature (Eq. 7). Table 3 gives the $b_0$-$b_2$ coefficients with $T_0=23^\circ$C and $γ_0$ fixed at 3.76nsec$^{-1}$.

Table 3. Polynomial coefficients of $D$ and $K$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
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<tbody>
<tr>
<td>$D$</td>
<td>GHz/mA</td>
<td>6.02</td>
<td>-2.58×10$^{-2}$</td>
<td>-1.60×10$^{-5}$</td>
</tr>
<tr>
<td>$K$</td>
<td>ns</td>
<td>0.40</td>
<td>-9.95×10$^{-5}$</td>
<td>1.37×10$^{-5}$</td>
</tr>
</tbody>
</table>

As shown in Figure 4, the modeled VCSEL small-signal transfer characteristics match well with the measured results, with an increased bias level resulting in both higher bandwidth and damping and an increased temperature yielding a reduction in bandwidth.

2.3 Large-signal dynamics

Finally, curve fitting of large-signal optical pulse responses is utilized to extract the remaining parameters for the rate equations which describe the VCSEL large-signal electro-optical dynamics.

\[
\frac{dN_c}{dt} = \frac{\eta(T)[I_a - I_a(T)]}{q} - \delta_g(T) \cdot \frac{P_c}{h\nu} \tag{12}
\]

\[
\frac{dP_c}{dt} = [\delta_g(T) - \delta_s(T)] \cdot \frac{P_c}{\tau_c} + P_{sp}(T), \tag{13}
\]

where $N_c$ is the total carriers in the active region and $P_c$ is the one-way recirculating power in the cavity. Eq. (12) and (13) are a reformulation of the more typical and detailed rate equations [22].

The round trip cavity gain $\delta_g$, spontaneous emission power $P_{sp}$, and round trip cavity time $\tau_c$ are

\[
\delta_g(T) = \frac{4g_0(T) \cdot L_a}{1 + \frac{4P_c}{P_{sat}(T)}} \ln \left( \frac{N}{N_tr} \right) \tag{14}
\]

\[
P_{sat}(T) = \frac{h\nu}{\tau_c} \cdot \beta \cdot V_a \cdot B(T) \cdot N^2 \tag{15}
\]

\[
\tau_c = m_c / v_0 \tag{16}
\]

where $g_0$ is the gain coefficient, $L_a$ is the length of the active region, $P_{sat}$ is the saturation power, $N=N_c/V_a$ is the carrier density, $V_a$ is the active region volume, $N_tr$ is the transparency carrier density, $\beta$ is the spontaneous capture efficiency, $B$ is the radiative recombination coefficient, $m_c$ is the round trip cavity delay normalized by the optical period $\tau_0=1/v_0$, and $v_0$ is the resonant cavity photon frequency. Note that the factor of four in Eq. (14) consists of a factor of two from the power traveling in both the forward and reverse directions and another factor of two is from the standing wave at the quantum wells.

All of these expressions contain parameters fixed by the VCSEL’s physical design, including
\[
\frac{m_v}{\lambda_0} = \frac{2n_{g} f_v}{2n_{g} \Delta \lambda} \]

and

\[
V_a = \frac{1}{4} \pi L_a d^2_{ox},
\]

where \( n_{g} \) is the average group index, \( n \) is the average refractive index, \( L_c \) is the cavity length, and \( d_{ox} \) is the oxide diameter. Parameters \( \delta, n_{g}, \nu_{o}, d_{ox}, \) and \( L_a \) come directly from the VCSEL design, while \( m_v \) is estimated by DBR design simulations that extract the the output wavelength shift with the cavity length change.

The round trip cavity loss \( \delta_c \) and threshold current \( I_{th} \) are expressed as

\[
\delta_c = \delta_1 + \delta_a \]

\[
I_a (T) = \frac{q \cdot V_a}{\eta(T)} \left\{ A \cdot N_{th}(T) + B(T) \left[ N_{th}(T) \right]^2 + C \left[ N_{th}(T) \right]^3 \right\}
\]

with

\[
N_{th} (T) = N_{th} e^{\frac{\delta}{\frac{T}{T_0}}},
\]

where \( N_{th} \) is the threshold carrier density, \( A \) is the non-radiative recombination coefficient, and \( C \) is the Auger recombination coefficient. Here the loss from the back mirror \( R_2 \) is negligible for typical VCSELS, but if needed can be included into \( \delta \). \( \eta \) and \( \delta_a \) are obtained from previous dc parameter extraction, while the \( A \) and \( C \) parameters are approximated as zero to simplify parameter extraction since their values are commonly quite small for VCSELS. This allows the \( B \) value to be resolved from Eq. (19-21) once the gain coefficient \( g_0 \) and the transparency carrier density \( N_{tr} \) are determined.

The gain coefficient \( g_0 \) and saturation power \( P_{sat} \) are described by

\[
g_o (T) = a_0 (T) \cdot N_{tr}
\]

\[
P_{sat} (T) = V_a \cdot V_h \cdot \nu_{g} / \varepsilon (T),
\]

where \( a_0 \) is the differential gain, \( v_{g} = c/n_{g} \) is the group velocity, and \( \varepsilon \) is the gain compression factor. As the differential gain \( a_0 \) and gain compression \( \varepsilon \) are related to the D- and K-factors, the VCSEL small-signal characteristics are utilized to determine their values.

\[
a_0 (T) = \frac{\pi^3 \cdot q \cdot \tau_p \cdot d_{ox} \cdot \left[ D(T) \right]^2}{2 \eta (T)}
\]

\[
\varepsilon (T) = v_{g} \cdot \tau_p \cdot a_0 (T) \left[ \frac{K(T)}{4 \pi^2 \cdot \tau_p} - 1 \right]
\]

with

\[
\tau_p = \tau_c / \delta_c,
\]

where \( \tau_p \) is the photon lifetime.

Parameters \( N_{tr}, \beta, \) and \( B \) are extracted by curve fitting the Figure 5 40ps optical output pulse responses with a 6mA swing centered at a 6mA bias level at 23°C. In order to describe the parameters’ thermal dependency, they are modeled as a second-order function of temperature (Eq. (7)). Table 4 gives the \( b_0-b_2 \) coefficients with \( T_0=23°C \). Comparing the modeled and measured Figure 5 high-speed large-signal pulse responses, the model accurately captures the reduction in pulse amplitude and change in relaxation oscillation behavior as the temperature changes from 23°C to 80°C.
Table 4. Rate equation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Description</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_c )</td>
<td>-</td>
<td>Round trip cavity loss</td>
<td>( 6.5 \times 10^{-3} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td>-</td>
<td>Output DBR transmissivity</td>
<td>( 3.8 \times 10^{-3} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( v_0 )</td>
<td>Hz</td>
<td>Photon frequency</td>
<td>( 3.0 \times 10^{14} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( n_g )</td>
<td>-</td>
<td>Group index</td>
<td>3.55</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( m_c )</td>
<td>-</td>
<td>Cavity mode parameter</td>
<td>8.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \eta_i )</td>
<td>-</td>
<td>Injection efficiency</td>
<td>0.90</td>
<td>-1.8 \times 10^{-3}</td>
<td>-8.5 \times 10^{-6}</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>cm(^2)</td>
<td>Differential gain</td>
<td>( 1.3 \times 10^{15} )</td>
<td>-8.3 \times 10^{-18}</td>
<td>8.7 \times 10^{-21}</td>
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<tr>
<td>( L_a )</td>
<td>cm</td>
<td>Active region length</td>
<td>( 1.8 \times 10^{-6} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( d_{ox} )</td>
<td>cm</td>
<td>Oxide diameter</td>
<td>( 7.0 \times 10^{-4} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( N_{tr} )</td>
<td>cm(^3)</td>
<td>Transparency carrier density</td>
<td>( 3.6 \times 10^{18} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>cm(^3)</td>
<td>Gain compression factor</td>
<td>( 6.8 \times 10^{19} )</td>
<td>-6.6 \times 10^{-21}</td>
<td>8.9 \times 10^{-24}</td>
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<tr>
<td>( \beta )</td>
<td>-</td>
<td>Spontaneous capture efficiency</td>
<td>( 4.0 \times 10^{-3} )</td>
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<td>0</td>
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<tr>
<td>( A )</td>
<td>s(^{-1})</td>
<td>Non-radiative recombination coefficient</td>
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<td>0</td>
</tr>
<tr>
<td>( B )</td>
<td>cm(^3)/s</td>
<td>Radiative recombination coefficient</td>
<td>( 1.3 \times 10^{10} )</td>
<td>-6.2 \times 10^{-13}</td>
<td>-1.3 \times 10^{-15}</td>
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<tr>
<td>( C )</td>
<td>cm(^3)/s</td>
<td>Auger recombination coefficient</td>
<td>0</td>
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<td>0</td>
</tr>
</tbody>
</table>

3. Model verification

Experimental verification of the model is performed at 25Gb/s over different bias and temperature conditions with the Figure 6 test setup. An arbitrary waveform generator (AWG) produces a 2\(^{15}\)-1 PRBS data pattern that is applied to the VCSEL through a bias tee for bias tuning. The AWG voltage swing is set to achieve ~5dB ER at 6mA bias current and 23°C substrate temperature, and is kept the same for all measurements. An oscilloscope with a 26GHz optical module is utilized to record the 25Gb/s eye diagrams. VCSEL performance over temperature is observed by placing the device under test (DUT) on a heater. Transition times of 23ps are used in all the modeling results, which matches the equipment used in the measurements.
As shown in the 25Gb/s eye diagrams of Figure 7, excellent matching is achieved between the simulated and measured results at bias currents of 4mA, 5mA, and 6mA for substrate temperatures of 23°C (left column) and 80°C (right column). Note that the same modeling parameters are used in all the simulations, with the coefficients automatically updated as the substrate temperature and bias conditions are varied for the different cases. While the 25Gb/s eye is open at 23°C for a low 4mA bias (Figure 7(a)), a relatively large rising-edge overshoot and deterministic jitter is observed both in the measurement and modeling results. For this bias condition, the performance degrades to an unacceptable level at 80°C (Figure 7(b)) due to the degraded slope efficiency and bandwidth. The model correctly captures the reduced jitter and overshoot for the eye diagrams with an increased 5mA bias. While a good eye opening is observed at 23°C (Figure 7(c)), again the performance degrades at 80°C (Figure 7(d)). Operating the device with a 6mA bias provides excellent eye opening at 23°C (Figure 7(e)) and adequate performance at 80°C (Figure 7(f)), with the model correctly displaying small overshoot and deterministic jitter at this bias level. Overall, the Figure 7 results show excellent correlation between the proposed VCSEL model and measurements over varying bias level and temperature.
4. Conclusion

Co-simulation environments which allow for the optimization of driver circuitry with accurate compact VCSEL models are necessary in order to enable efficient optical interconnect transceiver systems operating at data rates in excess of 20Gb/s. The presented compact comprehensive Verilog-A VCSEL model captures thermally-dependent electrical and optical dynamics and provides dc, small signal, and large-signal simulation capabilities. Model parameters are extracted utilizing dc, small-signal electrical and optical responses, and large-signal high-speed optical pulse responses over a set of bias and temperature conditions. Excellent matching between simulated and measured 25Gb/s eye diagrams at different bias currents and substrate temperatures is achieved.